

## Exercises – Sheet 2

Zürich, October 1, 2021

### Exercise 4

Provide an infinite sequence  $(x_n)_{n=1}^{\infty}$  of pairwise distinct words that satisfies the respective condition or prove that no such sequence exists.

- (a) There is a constant  $c \in \mathbb{N}$  such that, for all  $n \in \mathbb{N} - \{0\}$ , we have

$$K(x_n) \leq \log_2 \log_2 |x_n| + c.$$

- (b) There is a constant  $c \in \mathbb{N}$  such that, for all  $n \in \mathbb{N} - \{0\}$ , we have

$$K(x_n) \leq \log_2 \sqrt{n} + c.$$

**10 points**

### Exercise 5

Prove that the set  $\{n^2 \mid n \in \mathbb{N}\}$  contains only finitely many numbers that can be considered random.

**10 points**

### Exercise 6

We consider the language

$$L = \{1^i 0^j 1^k \mid i, j, k \in \mathbb{N} - \{0\}\}.$$

Let  $x_n$  be the  $n$ -th word in  $L$  with respect to the canonical order. Prove that there is a constant  $c \in \mathbb{N}$  such that, for all  $n \in \mathbb{N} - \{0\}$ , we have

$$K(x_n) \leq 3 \cdot \log_2(|x_n|) + c.$$

**10 points**

**Submission:** On Friday, October 8, 2021, by 11:15 at the latest, as a legible PDF via e-mail directly to the respective teaching assistant.