ETH zürich
Departement Informatik

Theoretische Informatik

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https://courses.ite.inf.ethz.ch/theo_inf_21

Exercises – Sheet 5

Zürich, October 22, 2021

Exercise 13

Prove that the following two languages are not regular.

(a) $L_1 = \{u\#v\#w \mid u, v, w \in \{0,1\}^+ \text{ and } \text{Number}(u) \cdot \text{Number}(v) = \text{Number}(w)\}$

(b) $L_2 = \{0^p \mid p \text{ is a prime number}\}$

10 points

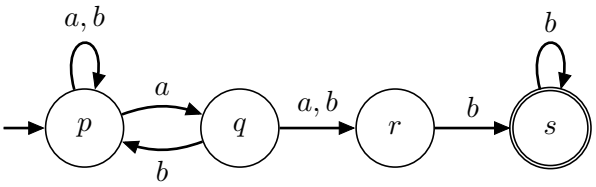
Exercise 14

(a) Design a nondeterministic finite automaton with at most 8 states for the language

$$L = \{x \in \{0,1\}^* \mid |x|_0 \bmod 3 = 0 \text{ or } x = 1y01 \text{ for } y \in \{0,1\}^+\} .$$

Provide a graphic representation of your automaton and explain the ideas underlying its design.

(b) Use the power set construction to transform the following nondeterministic finite automaton into an equivalent deterministic finite automaton.



Provide a graphic representation of the constructed automaton. You can leave out all unreachable states.

10 points

(please turn over)

Exercise 15

Prove the following two statements by providing an explicit automaton construction and informally arguing about its correctness.

(a) Let L_1 and L_2 be two regular languages over the alphabet $\Sigma = \{a, b\}$. Then the language $L = L_1 \cdot \{c\} \cdot L_2$ over the alphabet $\{a, b, c\}$ is also regular.

(b) Let L be a regular language over an arbitrary alphabet Σ. Then the language $L^{\mathsf{R}} \coloneqq \{w^{\mathsf{R}} \mid w \in L\}$ is also regular.

10 points

Submission: On Friday, October 29, 2021, by 11:15 at the latest, as a legible PDF via e-mail directly to the respective teaching assistant.