ETHzürich
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Theoretische Informatik

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https://courses.ite.inf.ethz.ch/theo_inf_21

Exercises – Sheet 6

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Exercise 16

(a) We consider an infinite hotel Hilbert. All rooms in this hotel are booked and there are three infinite buses arriving with new guests. Some of the new guests have special requirements: the guests from the bus 1 insist on odd room numbers and every guest from the bus 2 insists on having at least one neighbour who has also arrived on the bus 2. Describe a strategy for the porter to accommodate all guests and meet their requirements.

(b) We again consider the hotel Hilbert and now we assume that it is completely empty. Because a nightly move is very inconvenient for the guests, we seek to not move any guests in the following. We suppose that at an arbitrary countable number of points in time a group with a finite or countably infinite amount of guests arrives. The guests are numbered within each group. Describe a strategy for the porter to accommodate all guests so that no guest ever has to move to a different room.

10 points

Exercise 17

Let w_i be the i-th word over Σ_{bool} with respect to the canonical order and let M_i be the i-th Turing machine in the canonical order. We consider the languages

$$L_1 = \{w \in \{0,1\}^* \mid w = w_i \text{ for some } i \in \mathbb{N} - \{0\} \text{ and } M_{2i} \text{ does not accept } w\}$$

and

$$L_2 = \{w \in \{0,1\}^* \mid w = w_i \text{ for some } i \in \mathbb{N} - \{0\} \text{ and } M_{\lceil i/2 \rceil} \text{ does not accept } w\}.$$

Prove that one of the two languages L_1 and L_2 is not recursively enumerable and explain why an analogous proof does not work for the other language.

10 points

Exercise 18

In this exercise, we define an alternative model of Turing machines that differs from our standard model in the following two aspects:

(i) Besides an accepting and rejecting state, there exists exactly one additional state.

(ii) In a step, the read/write head can not only read and update the symbol in the square of the tape on which it is adjusted, but also the symbols in the neighbouring squares of the tape.

(a) Define such a new model by formally providing a tuple together with the corresponding properties analogously to Definition 4.1 in the textbook and by describing the additional adjustments to the standard model to define the notions of a *configuration*, *initial configuration*, *step*, *computation*, and *accepted language*.

(b) Given a Turing machine M in the standard model, describe how an equivalent Turing machine M' in the new model can be constructed. Provide M' as a tuple, including a complete description of the transition function.

10 points