ETH zürich
Departement Informatik

Theoretische Informatik

Prof. Dr. Juraj Hromkovič
Dr. Hans-Joachim Böckenhauer
https://courses.ite.inf.ethz.ch/theo_inf_21

Exercises – Sheet 7

Zürich, November 5, 2021

Exercise 19

Prove the following two statements.

(a) $L_{\mathrm{H}}^{\complement}$ is not recursively enumerable.

(b) $L'_{\mathrm{diag}} \leq_{\mathrm{R}} L_{\mathrm{H}}$, where

$$L'_{\mathrm{diag}} = \{\, w \in \{0,1\}^* \mid w = w_{2i} \text{ for some } i \in \mathbb{N} \text{ and } M_i \text{ does not accept } w \,\}.$$

10 points

Exercise 20

Prove the following two statements by providing a concrete mapping reduction and proving its correctness.

(a) $L_{\mathrm{H}} \leq_{\mathrm{m}} L_{\mathrm{UU},\lambda}$, where $L_{\mathrm{UU},\lambda} = \{\mathrm{Kod}(M_1)\#\mathrm{Kod}(M_2) \mid M_1 \text{ and } M_2 \text{ accept } \lambda\}$,

(b) $L_{\mathrm{U}}^{\complement} \leq_{\mathrm{m}} L_{\mathrm{diag}}$.

10 points

Exercise 21

We consider the language

$$L_{\mathrm{union}} = \{\mathrm{Kod}(M)\#\mathrm{Kod}(M')\#w \mid w \in L(M) \cup L(M')\}\,.$$

Prove the following statements:

(a) $L_{\mathrm{union}} \in \mathcal{L}_{\mathrm{RE}}$,

(b) $L_{\mathrm{U}} \leq_{\mathrm{m}} L_{\mathrm{union}}$, and

(c) $L_{\mathrm{union}} \leq_{\mathrm{m}} L_{\mathrm{U}}$.

10 points

Submission: Due to the first midterm exam on November 11, we extend the submission deadline for this exercise sheet by one week. Hence, the deadline for submitting this sheet is on Friday, November 19, 2021, at 11:15. Exercise sheet 8 will be published on November 19.