

## Exercises – Sheet 8

Zürich, November 19, 2021

### Exercise 22

(a) We consider the language

$$L_{\text{all}} = \{\text{Kod}(M) \mid M \text{ is a Turing machine with } L(M) = \Sigma^*\}.$$

Show that  $L_{\text{H}}^c \leq_{\text{EE}} L_{\text{all}}$  by providing a concrete reduction and proving its correctness.

(b) We consider the language

$$L_{\text{infinite}} = \{\text{Kod}(M) \mid M \text{ halts on no input}\}.$$

Show that  $(L_{\text{infinite}})^c \in \mathcal{L}_{\text{RE}}$  holds.

**10 points**

### Exercise 23

Let  $M$  be a 1-tape Turing machine that always halts. Prove that there exists an equivalent 2-tape Turing machine  $A$  such that, for some constant  $c$  and for all  $n$ :

$$\text{Time}_A(n) \leq \frac{\text{Time}_M(n)}{2} + \frac{13n}{12} + c.$$

*Hint:* The 2-tape TM  $A$  can simulate 12 cells of the input or working tape of  $M$  in a single cell.

**10 points**

### Exercise 24

Design an unrestricted (also called type-0) grammar for the language

$$L = \{0^n 1^n 2^n \mid n \in \mathbb{N}\},$$

justify briefly your design, and provide a derivation of the word 000111222 in your grammar.

**10 points**

**Submission:** On Friday, November 26, 2021, by 11:15 at the latest, as a legible PDF via e-mail directly to the respective teaching assistant.