

Exercises – Sheet 9

Zürich, November 26, 2021

Exercise 25

- (a) Let X be a set and let $\mathcal{S} \subseteq \mathcal{P}(X)$ be such that $\bigcup_{S \in \mathcal{S}} S = X$. A subset $\mathcal{C} \subseteq \mathcal{S}$ is a *set cover* of X if $X = \bigcup_{S \in \mathcal{C}} S$. The set cover problem (SCP) is defined as

$$\text{SCP} = \{(X, \mathcal{S}, k) \mid X \text{ has a set cover } \mathcal{C} \subseteq \mathcal{S} \text{ such that } |\mathcal{C}| \leq k\}.$$

Prove that $\text{VC} \leq_p \text{SCP}$.

- (b) A *dominating set* of a graph $G = (V, E)$ is a set $D \subseteq V$ such that every vertex $v \in V$ satisfies the following: $v \in D$ or there exists some $w \in D$ such that $\{v, w\} \in E$. The dominating set problem (DS) is defined as

$$\text{DS} = \{(G, k) \mid G \text{ has a dominating set } D \text{ such that } |D| \leq k\}.$$

Prove that $\text{SCP} \leq_p \text{DS}$.

10 points

Exercise 26

Let E3SAT be the set of all formulas in CNF that have a satisfying assignment and whose clauses consist of exactly three literals of pairwise distinct variables each. Prove that $3\text{SAT} \leq_p \text{E3SAT}$.

10 points

Exercise 27

We consider the languages

$$\text{LARGE-CLIQUE} = \{(G, k) \mid G = (V, E) \text{ is an undirected graph that contains a } k\text{-clique of size } k \geq |V|/3\}$$

and

$$\text{VERY-LARGE-CLIQUE} = \{(G, k) \mid G = (V, E) \text{ is an undirected graph that contains a } k\text{-clique of size } k \geq |V| - 3\}.$$

For each of the two languages, prove that it is NP-complete or that it is in P. **10 points**

Submission: On Friday, December 3, 2021, by 11:15 at the latest, as a legible PDF via e-mail directly to the respective teaching assistant.